Chapter 5: Work, Power & Energy

“Ambition is like a vector; it needs magnitude and direction. Otherwise, it’s just energy.”
— Grace Lindsay

Objectives

1. Define work and calculate the work done by a force.
2. Apply relationships between work, net force, displacement, velocity and kinetic energy to solve a variety of problems.
3. Calculate the power of a system.
4. Identify, describe, and calculate the potential energy of a system.
5. Apply conservation of energy to analyze energy transitions and transformations in a system.
6. Analyze the relationship between work done on or by a system, and the energy gained or lost by that system.
7. Use Hooke’s Law to determine the elastic force on an object.
8. Calculate a system’s elastic potential energy.
Work, energy and power are highly inter-related concepts that come up regularly in everyday life. You do work on an object when you move it. The rate at which you do the work is your power output. When you do work on an object, you transfer energy from one object to another. In this chapter you'll explore how energy is transferred and transformed, how doing work on an object changes its energy, and how quickly work can be done.

**Work**

Sometimes you work hard. Sometimes you’re a slacker. But, right now, are you doing work? And what is meant by the word “work?” In physics terms, **work** is the process of moving an object by applying a force, or, more formally, work is the energy transferred by an external force exerted on an object or system that moves the object or system.

I’m sure you can think up countless examples of work being done, but a few that spring to mind include pushing a snowblower to clear the driveway, pulling a sled up a hill with a rope, stacking boxes of books from the floor onto a shelf, and throwing a baseball from the pitcher’s mound to home plate.

Let’s take a look at a few scenarios and investigate what work is being done.

In the first scenario, a monkey in a jet pack blasts through the atmosphere, accelerating to higher and higher speeds. In this case, the jet pack is applying a force causing it to move. But what is doing the work? Hot expanding gases are pushed backward out of the jet pack. Using Newton’s 3rd Law, you observe the reactionary force of the gas pushing the jet pack forward, causing a displacement. Therefore, the expanding exhaust gas is doing work on the jet pack.

In the second scenario, a girl struggles to push her stalled car, but can’t make it move. Even though she’s expending significant effort, no work is being done on the car because it isn’t moving.

In the final scenario, a child in a ghost costume carries a bag of Halloween candy across the yard. In this situation, the child applies a force upward on the bag, but the bag moves horizontally. From this perspective, the forces of the child’s arms on the bag don’t cause the displacement, therefore no work is being done by the child.

It’s important to note that when calculating work, only the force applied in the direction of the object’s displacement counts! This means that if the force and displacement vectors aren’t in exactly the same direction, you need to take the component of force in the direction of the object’s displacement.
To do this, line up the force and displacement vectors tail-to-tail and measure the angle between them. Since this component of force can be calculated by multiplying the force by the cosine of the angle between the force and displacement vectors, you can write the work equation as:

\[ W = \Delta E = F_r r = Fr \cos \theta \]

\( W \) is the work done, \( F \) is the force applied in newtons, \( r \) is the object’s displacement in meters, and \( \theta \) is the angle between \( F \) and \( r \). The equation as a whole states that the work done is equal to the energy transferred, which is equal to the component of force parallel to the displacement multiplied by the displacement, which is equal to the magnitude of the force, multiplied by the magnitude of the displacement, multiplied by the cosine of the angle between them. When the work done on an object is independent of the object’s path, the force doing the work is known as a **conservative force**.

The units of work can be found by performing unit analysis on the work formula. If work is force multiplied by distance, the units must be the units of force multiplied by the units of distance, or newtons multiplied by meters. A newton-meter is also known as a Joule (J).

5.01 Q: An appliance salesman pushes a refrigerator 2 meters across the floor by applying a force of 200N. Find the work done.

5.01 A: Since the force and displacement are in the same direction, the angle between them is 0.

\[ W = F_r r = Fr \cos \theta = (200N)(2m)\cos0 = 400J \]

5.02 Q: A friend’s car is stuck on the ice. You push down on the car to provide more friction for the tires (by way of increasing the normal force), allowing the car’s tires to propel it forward 5m onto less slippery ground. How much work did you do?

5.02 A: You applied a downward force, yet the car’s displacement was sideways. Therefore, the angle between the force and displacement vectors is 90°.

\[ W = F_r r = Fr \cos \theta = Fr \cos90 = 0 \]
**5.03 Q:** You push a crate up a ramp with a force of 10N. Despite your pushing, however, the crate slides down the ramp a distance of 4m. How much work did you do?

**5.03 A:** Since the direction of the force you applied is opposite the direction of the crate’s displacement, the angle between the two vectors is 180°.

\[
W = F_r r = Fr \cos \theta = (10N)(4m)\cos 180\degree = -40J
\]

**5.04 Q:** How much work is done in lifting an 8-kg box from the floor to a height of 2m above the floor?

**5.04 A:** It’s easy to see the displacement is 2m, and the force must be applied in the direction of the displacement, but what is the force? To lift the box you must match and overcome the force of gravity on the box. Therefore, the force applied is equal to the gravitational force, or weight, of the box, \( mg=(8\text{kg})(9.8\text{m/s}^2)=78.4\text{N} \).

\[
W = F_r r = Fr \cos \theta = (78.4N)(2m)\cos 0\degree = 157J
\]

**5.05 Q:** Barry and Sidney pull a 30-kg wagon with a force of 500N a distance of 20m. The force acts at a 30° angle to the horizontal. Calculate the work done.

**5.05 A:**

\[
W = F_r r = Fr \cos \theta = (500N)(20m)\cos 30\degree = 8660J
\]

**5.06 Q:** The work done in lifting an apple one meter near Earth’s surface is approximately

(A) 1 J
(B) 0.01 J
(C) 100 J
(D) 1000 J

**5.06 A:** (A) The trick in this problem is recalling the approximate weight of an apple. With an “order-of-magnitude” estimate, you can say an apple has a mass of 0.1 kg, or a weight of 1 N. Given this information, the work done is:

\[
W = F_r r = Fr \cos \theta = (1N)(1m)\cos 0\degree = 1J
\]
5.07 Q: As shown in the diagram, a child applies a constant 20-newton force along the handle of a wagon which makes a 25° angle with the horizontal.

How much work does the child do in moving the wagon a horizontal distance of 4.0 meters?

(A) 5.0 J
(B) 34 J
(C) 73 J
(D) 80 J

5.07 A: (D) \[ W = F_r = Fr \cos \theta = (20N)(4m)\cos 25^\circ = 73J \]

The area under a force vs. displacement graph is the work done by the force. Consider the situation of a block being pulled across a table with a constant force of 5 Newtons over a displacement of 5 meters, then the force gradually tapers off over the next 5 meters.

The work done by the force moving the block can be calculated by taking the area under the force vs. displacement graph (a combination of a rectangle and triangle) as follows:

\[ \text{Work} = \text{Area}_{\text{rectangle}} + \text{Area}_{\text{triangle}} \]
\[ \text{Work} = lw + \frac{1}{2}bh \]
\[ \text{Work} = (5m)(5N) + \frac{1}{2}(5m)(5N) \]
\[ \text{Work} = 37.5J \]
5.08 Q: A boy pushes his wagon at constant speed along a level sidewalk. The graph below represents the relationship between the horizontal force exerted by the boy and the distance the wagon moves.

What is the total work done by the boy in pushing the wagon 4.0 meters?
(A) 5.0 J
(B) 7.5 J
(C) 120 J
(D) 180 J

5.08 A: (C) 120 J \[ \text{Work} = \text{Area}_{\text{rectangle}} = lw = (4m)(30N) = 120J \]

5.09 Q: A box is wheeled to the right with a varying horizontal force. The graph below represents the relationship between the applied force and the distance the box moves.

What is the total work done in moving the box 6 meters?
(A) 9.0 J
(B) 18 J
(C) 27 J
(D) 36 J
5.09 A: \[ \text{Work} = \text{Area}_{\text{rectangle}} + \text{Area}_{\text{triangle}} \]
\[ \text{Work} = lw + \frac{1}{2}bh \]
\[ \text{Work} = (3m)(6N) + \frac{1}{2}(3m)(6N) \]
\[ \text{Work} = 27J \]

5.10 Q: An 80-kg wooden box is pulled 10 meters horizontally across a wood floor at a constant velocity by a 250-newton force at an angle of 37 degrees above the horizontal. If the coefficient of kinetic friction between the floor and the box is 0.315, find the work done by friction.

5.10 A: First, draw a diagram of the situation.

Next, create a FBD and pseudo-FBD detailing the forces on the box.

Then, write a Newton’s 2nd Law equation in the x-direction and solve for the force of friction, recognizing that the acceleration of the box must be zero since it moves at constant velocity.
\[ F_{\text{net}} = F_{\text{app}} \cos \theta - f = ma \]
\[ \rightarrow \]
\[ a = 0 \]
\[ 250N \cos(37^\circ) - f = 0 \rightarrow f = 200N \]

Finally, solve for the work done by friction.
\[ W_f = F_r \cos \theta = (200N)(10m) \cos(180^\circ) = -2000J \]
5.11 Q: Four carts, initially at rest on a flat surface, are subjected to varying forces as the carts move to the right a set distance, depicted in the diagram below.

Rank the four carts from least to greatest in terms of
I) work done by the applied force on the carts
II) inertia
III) normal force applied by the surface to the carts

5.11 A: I) D, A, C, B
II) A, D, C, B
III) A, D, C, B

Power

Power is a term used quite regularly in all aspects of life. People talk about how powerful the new boat motor is, the power of positive thinking, and even the power company’s latest bill. All of these uses of the term power relate to how much work can be done in some amount of time, or the rate at which energy is transferred.

In physics, work can be defined in two ways. Work is the process of moving an object by applying a force. The rate at which the force does work is known as power \( P \). Work is also the transfer of energy, so power is also the rate at which energy is transferred into, out of, or within a system. The units of power are the units of work divided by time, or Joules per second, known as a Watt \( W \).

\[
P = \frac{W}{t} = \frac{\Delta E}{t}
\]

Since power is the rate at which work is done, it is possible to have the same amount of work done but with a different supplied power, if the time is different.
5.12 Q: Rob and Peter move a sofa 3 meters across the floor by applying a combined force of 200N horizontally. If it takes them 6 seconds to move the sofa, what amount of power did they supply?

5.12 A: \[ P = \frac{W}{t} = \frac{Fr \cos \theta}{t} = \frac{(200\text{N})(3\text{m})}{6\text{s}} = 100\text{W} \]

5.13 Q: Kevin then pushes the same sofa 3 meters across the floor by applying a force of 200N. Kevin, however, takes 12 seconds to push the sofa. What amount of power did Kevin supply?

5.13 A: \[ P = \frac{W}{t} = \frac{Fr \cos \theta}{t} = \frac{(200\text{N})(3\text{m})}{12\text{s}} = 50\text{W} \]

As you can see, although Kevin did the same amount of work as Rob and Peter in pushing the sofa (600J), Rob and Peter supplied twice the power of Kevin because they did the same work in half the time!

There’s more to the story, however. Since power is defined as work over time, and because work is equal to force (in the direction parallel to the displacement) multiplied by displacement, you can replace work in the equation with \( F \times r \times \cos(\theta) \):

\[ P = \frac{W}{t} = \frac{Fr \cos \theta}{t} \]

Looking carefully at this equation, you can observe a displacement divided by time. Since displacement divided by time is the definition of average velocity, you can replace \( \Delta r / t \) with \( v \) in the equation and, assuming the force is in the direction of the displacement (\( \cos \theta = 1 \)) you obtain:

\[ P = \frac{W}{t} = \frac{Fr \cos \theta}{t} = Fv \]

So, not only is power equal to work done divided by the time required, it’s also equal to the force applied (in the direction of the displacement) multiplied by the average velocity of the object.

5.14 Q: Motor A lifts a 5000N steel crossbar upward at a constant 2 m/s. Motor B lifts a 4000N steel support upward at a constant 3 m/s. Which motor is supplying more power?
5.14 A: Motor B supplies more power than Motor A.
\[ P_{\text{MotorA}} = FV = (5000 \text{ N})(2 \text{ m/s}) = 10000 \text{ W} \]
\[ P_{\text{MotorB}} = FV = (4000 \text{ N})(3 \text{ m/s}) = 12000 \text{ W} \]

5.15 Q: A 70-kilogram cyclist develops 210 watts of power while pedaling at a constant velocity of 7 meters per second east. What average force is exerted eastward on the bicycle to maintain this constant speed?
(A) 490 N
(B) 30 N
(C) 3.0 N
(D) 0 N

5.15 A: (B) \[ P = Fv \]
\[ F = \frac{P}{v} = \frac{210 \text{ W}}{7 \text{ m/s}} = 30 \text{ N} \]

5.16 Q: Alien A lifts a 500-newton child from the floor to a height of 0.40 meters in 2 seconds. Alien B lifts a 400-newton student from the floor to a height of 0.50 meters in 1 second. Compared to Alien A, Alien B does
(A) the same work but develops more power
(B) the same work but develops less power
(C) more work but develops less power
(D) less work but develops more power

5.16 A: (A) the same work but develops more power.

5.17 Q: A 110-kilogram bodybuilder and his 55-kilogram friend run up identical flights of stairs. The bodybuilder reaches the top in 4.0 seconds while his friend takes 2.0 seconds. Compared to the power developed by the bodybuilder while running up the stairs, the power developed by his friend is
(A) the same
(B) twice as much
(C) half as much
(D) four times as much

5.17 A: (A) the same.
5.18 Q: Mary holds a 5-kg mirror against the wall 1.5 meters above the ground for 20 seconds while Bob nails it in place. What is Mary’s power output during that time period?
(A) 2.45 Watts
(B) 3.68 Watts
(C) 66.7 Watts
(D) None of the above

5.18 A: (D) There is no power output because the work done is zero. There is no displacement of the mirror.

5.19 Q: Which of the following are appropriate units for power? Choose all that apply.
(A) \( \frac{J}{s} \)
(B) \( \frac{N \cdot m^2}{s^2} \)
(C) \( \frac{kg \cdot m^2}{s^2} \)
(D) \( \frac{kg \cdot m^2}{s^3} \)

5.19 A: (A) and (D) are appropriate units for power.

5.20 Q: A box of mass \( m \) is pushed up a ramp at constant velocity \( v \) to a maximum height \( h \) in time \( t \) by force \( F \). The ramp makes an angle of \( \theta \) with the horizontal as shown in the diagram below.

What is the power supplied by the force? Choose all that apply.
(A) \( \frac{mgh}{t} \)
(B) \( \frac{mg h}{t \sin \theta} \)
(C) \( \frac{Fh}{t \sin \theta} \)
(D) \( Fv \)

5.20 A: (C) and (D) are both correct expressions for the power supplied by the force.
Energy

We’ve all had days where we’ve had varying amounts of energy. You’ve gotten up in the morning, had to drag yourself out of bed, force yourself to get ready for school, and once you finally get to class, you don’t have the energy to do much work. Other days, when you’ve had more energy, you may have woken up before the alarm clock, hustled to get ready for the day while a bunch of thoughts bounced around in your head, and hurried on to begin your activities. Then, throughout the day, the more work you do, the more energy you lose... What’s the difference in these days?

In physics, energy is the ability or capacity to do work. And as mentioned previously, work is the process of moving an object. So, if you combine the definitions, energy is the ability or capacity to move an object. So far you’ve examined kinetic energy, or energy of motion, and therefore kinetic energy must be the ability or capacity of a moving object to move another object! Mathematically, kinetic energy is calculated using the formula:

\[ K = \frac{1}{2}mv^2 \]

Of course, there are more types of energy than just kinetic. Energy comes in many forms, which you can classify as kinetic (energy of motion) or potential (stored) to various degrees. This includes solar energy, thermal energy, gravitational potential energy, nuclear energy, chemical potential energy, sound energy, electrical energy, elastic potential energy, light energy, and so on. In all cases, energy can be transformed from one type to another and you can transfer energy from one object to another by doing work.
The units of energy are the same as the units of work, joules (J). Through dimensional analysis, observe that the units of KE (kg·m²/s²) must be equal to the units of work (N·m):

\[
\frac{kg \cdot m^2}{s^2} = N \cdot m = J
\]

**Gravitational Potential Energy**

**Potential energy** is energy an object possesses due to its position or condition. In order for an object to have potential energy, it must interact with another object or system. An object in isolation can have only kinetic energy.

**Gravitational potential energy** is the energy an object possesses because of its position in a gravitational field (height), where another object or system is providing the gravitational field.

Assume a 10-kilogram box sits on the floor. You can arbitrarily call its current potential energy zero, just to give a reference point. If you do work to lift the box one meter off the floor, you need to overcome the force of gravity on the box (its weight) over a distance of one meter. Therefore, the work you do on the box can be obtained from:

\[
W = F_r = (mg)\Delta y = (10 kg)(9.8 m/s^2)(1 m) = 98 J
\]

So, to raise the box to a height of 1m, you must do 98 Joules of work on the box. The work done in lifting the box is equal to the change in the potential energy of the box, so the box’s gravitational potential energy must increase by 98 Joules.

When you performed work on the box, you transferred some of your stored energy to the box. Along the way, it just so happens that you derived the formula for the change in gravitational potential energy of an object. In a constant gravitational field, the change in the object’s gravitational potential energy, \( \Delta U_g \), is equal to the force of gravity on the box (mg) multiplied by its change in height, \( \Delta y \).

\[
\Delta U_g = mg\Delta y
\]

This formula can be used to solve a variety of problems involving the potential energy of an object.
5.21 Q: The diagram below represents a 155-newton box on a ramp. Applied force F causes the box to slide from point A to point B.

What is the total amount of gravitational potential energy gained by the box?
(A) 28.4 J
(B) 279 J
(C) 868 J
(D) 2740 J

5.21 A: (B) $\Delta U_g = mg\Delta y = (155N)(1.8m) = 279J$

5.22 Q: Which situation describes a system with decreasing gravitational potential energy?
(A) a girl stretching a horizontal spring
(B) a bicyclist riding up a steep hill
(C) a rocket rising vertically from Earth
(D) a boy jumping down from a tree limb

5.22 A: (D) The boy’s height above ground is decreasing, so his gravitational potential energy is decreasing.

5.23 Q: Which is an SI unit for energy?
(A) $\frac{kg \cdot m^2}{s^2}$
(B) $\frac{kg \cdot m^2}{s}$
(C) $\frac{kg}{s}$
(D) $\frac{kg \cdot m}{s^2}$

5.23 A: (A) is equivalent to a newton-meter, also known as a Joule.
5.24 Q: A car travels at constant speed \( v \) up a hill from point A to point B, as shown in the diagram below.

![Diagram of a car traveling up a hill](image)

As the car travels from A to B, its gravitational potential energy (A) increases and its kinetic energy decreases  
(B) increases and its kinetic energy remains the same  
(C) remains the same and its kinetic energy decreases  
(D) remains the same and its kinetic energy remains the same

5.24 A: (B) The car’s height above ground increases so gravitational potential energy increases, and velocity remains constant, so kinetic energy remains the same. Note that the car’s engine must do work to maintain a constant velocity.

5.25 Q: An object is thrown vertically upward. Which pair of graphs best represents the object’s kinetic energy and gravitational potential energy as functions of its displacement while it rises?

![Graphs of kinetic and potential energy](image)

5.25 A: (2) shows the object’s kinetic energy decreasing as it slows down on its way upward, while its potential energy increases as its height increases.

5.26 Q: While riding a chairlift, a 55-kilogram snowboarder is raised a vertical distance of 370 meters. What is the total change in the snowboarder’s gravitational potential energy?  
(A) \( 5.4 \times 10^1 \) J  
(B) \( 5.4 \times 10^2 \) J  
(C) \( 2.0 \times 10^4 \) J  
(D) \( 2.0 \times 10^5 \) J
5.26 A:  \( 2.0 \times 10^5 \) J
\[ \Delta U_g = mg\Delta y = (55\text{kg})(9.8\text{m/s}^2)(370\text{m}) = 2 \times 10^5 \text{ J} \]

5.27 Q:  A pendulum of mass M swings on a light string of length L as shown in the diagram at right. If the mass hanging directly down is set as the zero point of gravitational potential energy, find the gravitational potential energy of the pendulum as a function of \( \theta \) and L.

5.27 A:  First use some basic geometry and trigonometry to analyze the situation and set up the problem. Redraw the diagram as shown at right, then, solve for the change in vertical displacement, \( \Delta y \).

\[ \cos \theta = \frac{adj}{hyp} \rightarrow adj = hyp \cos \theta = L \cos \theta \]

\[ \Delta y = L - adj = L - L \cos \theta = L(1 - \cos \theta) \]

Now, utilize your value for \( \Delta y \) to find the gravitational potential energy above your reference point.

\[ \Delta U_g = mg\Delta y = mgL(1 - \cos \theta) \]

Springs and Hooke’s Law

An interesting application of work combined with the Force and Displacement graph is examining the force applied by a spring. The more you stretch a spring, the greater the force of the spring. Similarly, the more you compress a spring, the greater the force. This can be modeled as a linear relationship known as Hooke’s Law, where the magnitude of the force applied by the spring is equal to a constant multiplied by the magnitude of the displacement of the spring.

\[ |\vec{F}_s| = k|x| \]

\( F_s \) is the force of the spring in newtons, \( x \) is the displacement of the spring from its equilibrium (or rest) position, in meters, and \( k \) is the spring constant, which tells you how stiff or powerful a spring is, in newtons per meter. The larger the spring constant, \( k \), the more force the spring applies per amount of displacement. In some cases you may see this written as \( F_s = -kx \), where the negative sign indicates the force is in the opposite direction of the displacement.
You can determine the spring constant of a spring by making a graph of the force from a spring on the y-axis, and placing the displacement of the spring from its equilibrium, or rest position, on the x-axis. The slope of the graph will give you the spring constant. For the case of the spring depicted in the graph at right, you can find the spring constant as follows:

\[ k = \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta F}{\Delta x} = \frac{20 \text{N} - 0 \text{N}}{0.1 \text{m} - 0 \text{m}} = 200 \text{ N/m} \]

You must have done work to compress or stretch the spring, since you applied a force and caused a displacement. You can find the work done in stretching or compressing a spring by taking the area under the graph. For the spring shown, to displace the spring 0.1m, you can find the work done as shown below:

\[ \text{Work} = \text{Area}_{\text{tri}} = \frac{1}{2}bh = \frac{1}{2}(0.1 \text{m})(20 \text{N}) = 1 \text{J} \]

5.28 Q: In an experiment, a student applied various forces to a spring and measured the spring’s corresponding elongation. The table at right shows his data.

<table>
<thead>
<tr>
<th>Force (newtons)</th>
<th>Elongation (meters)</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>1.0</td>
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<td>6.0</td>
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</table>

Plot force versus elongation and draw the best-fit line. Then, using your graph, calculate the spring constant of the spring. Show all your work.

5.28 A:

\[ k = \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta F}{\Delta x} = \frac{6 \text{N} - 0 \text{N}}{1.5 \text{m} - 0 \text{m}} = 4 \text{ N/m} \]
5.29 Q: In a laboratory investigation, a student applied various downward forces to a vertical spring. The applied forces and the corresponding elongations of the spring from its equilibrium position are recorded in the data table.

Construct a graph, marking an appropriate scale on the axis labeled “Force (N).” Plot the data points for force versus elongation. Draw the best-fit line or curve. Then, using your graph, calculate the spring constant of this spring. Show all your work.

<table>
<thead>
<tr>
<th>Force (newtons)</th>
<th>Elongation (meters)</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
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<td>2.5</td>
<td>0.046</td>
</tr>
</tbody>
</table>

5.29 A:

\[ k = \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta F}{\Delta x} = \frac{2.5N - 0.8N}{0.046m - 0.015m} = 55 \text{ N/m} \]

5.30 Q: A 10-newton force compresses a spring 0.25 meter from its equilibrium position. Calculate the spring constant of this spring.

5.30 A:

\[ |\vec{F}_s| = k|\vec{x}| \rightarrow k = \frac{|\vec{F}_s|}{|\vec{x}|} = \frac{10N}{0.25m} = 40 \text{ N/m} \]

Another form of potential energy involves the stored energy an object possesses due to its position in a stressed elastic system. An object at the end of a compressed spring, for example, has elastic potential energy. When the spring is released, the elastic potential energy of the spring will do work on the object, moving the object and transferring the energy of the spring into kinetic energy of the object. Other examples of elastic potential energy include tennis rackets, rubber bands, bows (as in bows and arrows), trampolines, bouncy balls, and even pole-vaulting poles.
The most common problems involving elastic potential energy in introductory physics involve the energy stored in a spring. As you learned previously, the force needed to compress or stretch a spring from its equilibrium position increases linearly. The more you stretch or compress the spring, the more force it applies trying to restore itself to its equilibrium position. This can be modeled using Hooke’s Law:

\[ |\vec{F}_s| = k|x| \]

Further, you can find the work done in compressing or stretching the spring by taking the area under a force vs. displacement graph for the spring.

\[ W = F \cdot \Delta \text{Area} = \frac{1}{2} \text{base} \cdot \text{height} = \frac{1}{2} (x)(kx) = \frac{1}{2} kx^2 \]

Since the work done in compressing or stretching the spring from its equilibrium position transfers energy to the spring, you can conclude that the potential energy stored in the spring must be equal to the work done to compress the spring. The potential energy of a spring \( U_s \) is therefore given by:

\[ U_s = \frac{1}{2} kx^2 \]

5.31 Q: A spring with a spring constant of 4.0 newtons per meter is compressed by a force of 1.2 newtons. What is the total elastic potential energy stored in this compressed spring?

(A) 0.18 J
(B) 0.36 J
(C) 0.60 J
(D) 4.8 J

5.31 A: (A) Us can’t be calculated directly since \( x \) isn’t known, but \( x \) can be found from Hooke’s Law:

\[ |\vec{F}_s| = k|x| \rightarrow |x| = \frac{|\vec{F}_s|}{k} = \frac{1.2 \text{ N}}{4 \text{ N/m}} = 0.3 \text{ m} \]

With \( x \) known, the potential energy equation for a spring can be utilized.

\[ U_s = \frac{1}{2} kx^2 = \frac{1}{2} (4 \text{ N/m})(0.3 \text{ m})^2 = 0.18 \text{ J} \]
5.32 Q: An unstretched spring has a length of 10 centimeters. When the spring is stretched by a force of 16 newtons, its length is increased to 18 centimeters. What is the spring constant of this spring?

(A) 0.89 N/cm  
(B) 2.0 N/cm  
(C) 1.6 N/cm  
(D) 1.8 N/cm

5.32 A: (B) \[ F_s \| \vec{x} \| = k \| \vec{x} \| \rightarrow k = \frac{F_s}{\| \vec{x} \|} = \frac{16 N}{8 \text{ cm}} = 2.0 \text{ N/cm} \]

5.33 Q: Which graph best represents the relationship between the elastic potential energy stored in a spring and its elongation from equilibrium?

5.33 A: (2) due to the displacement$^2$ relationship.

5.34 Q: A pop-up toy has a mass of 0.020 kilogram and a spring constant of 150 newtons per meter. A force is applied to the toy to compress the spring 0.050 meter.
(A) Calculate the potential energy stored in the compressed spring.
(B) The toy is activated and all the compressed spring’s potential energy is converted to gravitational potential energy. Calculate the maximum vertical height to which the toy is propelled.

5.34 A:  
(A) \[ U_k = \frac{1}{2} k x^2 = \frac{1}{2} (150 \text{ N/m})(0.05 \text{ m})^2 = 0.1875 \text{ J} \]

(B) \[ U_g = mg \Delta y \rightarrow \Delta y = \frac{U_g}{mg} = \frac{0.1875 \text{ J}}{(0.02 \text{ kg})(9.8 \text{ m/s}^2)} = 0.96 \text{ m} \]

5.35 Q:  A spring with a spring constant of 80 newtons per meter is displaced 0.30 meter from its equilibrium position. The potential energy stored in the spring is
(A) 3.6 J
(B) 7.2 J
(C) 12 J
(D) 24 J

5.35 A:  
(A) \[ U_s = \frac{1}{2} k x^2 = \frac{1}{2} (80 \text{ N/m})(0.3 \text{ m})^2 = 3.6 \text{ J} \]

Work-Energy Theorem

Of course, there are many different kinds of energy which haven’t been mentioned specifically. Energy can be converted among its many different forms, such as mechanical (which is kinetic, gravitational potential, and elastic potential), electromagnetic, nuclear, and thermal (or internal) energy.

When a force does work on a system, the work done changes the system’s energy. If the work done increases motion, there is an increase in the system’s kinetic energy. If the work done increases the object’s height, there is an increase in the system’s gravitational potential energy. If the work done compresses a spring, there is an increase in the system’s elastic potential energy. If the work is done against friction, however, where does the energy go? In this case, the energy isn’t lost, but instead increases the rate at which molecules in the object vibrate, increasing the object’s temperature, or internal energy.

The internal energy of a system includes the kinetic energy of the objects that make up the system and the potential energy of the configuration of the objects that make up the system.
The understanding that the work done on a system by an external force changes the energy of the system is known as the Work-Energy Relationship. If an external force does positive work on the system, the system’s total energy increases. If, instead, the system does work, the system’s total energy decreases. Put another way, you add energy to a system by doing work on it and take energy from a system when the system does the work (much like you add value to your bank account by making a deposit and take value from your account by writing a check).

When the force applied on an object is parallel to the object’s displacement, the work done increases the kinetic energy of the object. When the force applied is opposite the direction of the object’s displacement, the work done decreases the kinetic energy of the object. This is known as the Work-Energy Theorem.

5.36 Q: Given the following sets of velocity and net force vectors for a given object, state whether you expect the kinetic energy of the object to increase, decrease, or remain the same.

(A) \( \vec{v} \rightarrow \vec{F}_{\text{net}} \)

(B) \( \vec{v} \rightarrow \uparrow \)

(C) \( \vec{v} \rightarrow \downarrow \)

(D) \( \vec{v} \rightarrow \vec{F}_{\text{net}} \)

5.36 A: (A) decrease

(B) remain the same (no work done as long as \( v \) and \( F \) are perpendicular)

(C) increase

(D) decrease

5.37 Q: A chef pushes a 10-kilogram pastry cart from rest a distance of 5 meters with a constant horizontal force of 10 N. Assuming a frictionless surface, determine the cart’s change in kinetic energy and its final velocity.

5.37 A: First find the work done by the chef, which will be equal to the cart’s change in kinetic energy.

\[ W = F_{||}r = (10 \text{N})(5 \text{m}) = 50 \text{J} \]

Next, solve for the cart’s final velocity.

\[ K = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(50 \text{J})}{10 \text{kg}}} = 3.2 \text{ m/s} \]
5.38 Q: A pitcher throws a 143-gram baseball toward the catcher at 45 m/s. If the catcher’s hand moves back a distance of 6 cm in stopping the ball, determine the average force exerted on the catcher’s hand.

5.38 A: There are several paths to solving this problem, including application of the Work-Energy Theorem.

\[
W = \Delta K \rightarrow \vec{F} \cdot \vec{r} = \frac{1}{2}mv^2 \rightarrow F = \frac{mv^2}{2r} = \frac{(0.143 \text{ kg})(45 \text{ m/s})^2}{2(0.06 \text{ m})} = 2415 \text{ N}
\]

5.39 Q: In the following diagrams, a force \( \vec{F} \) acts on a cart in motion on a frictionless surface to change its velocity. The initial velocity of the cart and final velocity of each cart are shown. You do not know how far or in which direction the cart traveled. Rank the magnitude of the work done by the force on each cart from greatest to least.

5.39 A: C, B, A, D (First find the change in \( K \), which must equal the work done according to the Work-Energy Theorem.)

5.40 Q: Given the force vs. displacement graph below for a net force applied horizontally to an object of mass \( m \) initially at rest on a frictionless surface, determine the object’s final speed in terms of \( F_{\text{max}}, r_1, r_2, r_3, \) and \( m \). You may assume the force does not change its direction.
5.40 A: The work done is the area under the graph, which, according to the Work-Energy Theorem, is also equal to the change in the object’s kinetic energy, therefore you can set the area of the graph equal to the object’s final kinetic energy given the initial kinetic energy is zero.

\[ W = \text{Area} = \frac{1}{2}bh + lw + \frac{1}{2}bh = \frac{1}{2}mv^2 \rightarrow \]

\[ \frac{1}{2}F_{\text{max}} + (r_2 - r_1)F_{\text{max}} + \frac{1}{2}(r_3 - r_2)F_{\text{max}} = \frac{1}{2}mv^2 \rightarrow \]

\[ v^2 = \frac{F_{\text{max}}}{m}(r_3 + r_2 - r_1) \rightarrow v = \sqrt{\frac{F_{\text{max}}}{m}(r_3 + r_2 - r_1)} \]

Conservation of Energy

“Energy cannot be created or destroyed... it can only be changed.”

Chances are you’ve heard that phrase before. It’s one of the most important concepts in all of physics. It doesn’t mean that an object can’t lose energy or gain energy. What it means is that energy can be changed into different forms, and transferred from system to system, but it never magically disappears or reappears. In the world of physics, you can never truly destroy energy. The understanding that the total amount of energy in the universe remains fixed is known as the law of conservation of energy.

Objects and systems can possess multiple types of energy. The energy of a system includes its kinetic energy, potential energy, and its internal energy. Mechanical energy is the sum of an object’s kinetic energy as well as its gravitational potential and elastic potential energies. Non-mechanical energy forms include chemical potential, nuclear, and thermal.

Total energy is always conserved in any closed system, which is the law of conservation of energy. By confining the discussion to just the mechanical forms of energy, however, if you neglect the effects of friction you can also state that total mechanical energy is constant in any system.

Take the example of an F/A-18 Hornet jet fighter with a mass of 20,000 kilograms flying at an altitude of 10,000 meters above the surface of the earth with a velocity of 250 m/s. In this scenario, you can calculate the total mechanical energy of the jet fighter as follows by assuming ground level is gravitational potential energy level zero:

\[ E_f = \Delta U_g + K = mg\Delta y + \frac{1}{2}mv^2 \]

\[ E_f = (20000\text{kg})(9.8 \text{ m/s}^2)(10000\text{m}) + \frac{1}{2}(20000\text{kg})(250 \text{ m/s})^2 \]

\[ E_f = 2.59 \times 10^8 \text{ J} \]
Now, assume the Hornet dives down to an altitude of 2,000 meters above the surface of the Earth. Total mechanical energy remains constant, and the gravitational potential energy of the fighter decreases, therefore the kinetic energy of the fighter must increase. The fighter’s velocity goes up as a result of flying closer to the Earth! For this reason, a key concept in successful dogfighting taught to military pilots is that of energy conservation!

You can even calculate the new velocity of the fighter jet since you know its new height and its total mechanical energy must remain constant. Solving for velocity, you find that the Hornet has almost doubled its speed by “trading in” 8000 meters of altitude for velocity!

\[
E_T = \Delta U_g + K = mg\Delta y + \frac{1}{2}mv^2 \\
\frac{1}{2}mv^2 = E_T - mg\Delta y \\
v = \sqrt{\frac{2(E_T - mg\Delta y)}{m}} \\
v = \sqrt{\frac{2(2.59 \times 10^7 J - (20000 kg)(9.8 \ m/s^2)(2000 m))}{20000 kg}} = 469 \ m/s
\]

If instead you had been told that some of the mechanical energy of the jet was lost to air resistance (friction), you could also account for that by stating that the total mechanical energy of the system is equal to the gravitational potential energy, the kinetic energy, and the change in internal energy of the system (Q). This leads to the conservation of mechanical energy formula:

\[
E_T = U + K + Q
\]

Let’s take another look at free fall, only this time, you can analyze a falling object using the law of conservation of energy and compare it to the analysis using the kinematic equations studied previously.

The problem: An object falls from a height of 10m above the ground. Neglecting air resistance, find its velocity the moment before the object strikes the ground.

**Conservation of Energy Approach:** The energy of the object at its highest point must equal the energy of the object at its lowest point, therefore:

\[
E_{\text{top}} = E_{\text{bottom}} \\
U_{\text{top}} = K_{\text{bottom}} \\
mg\Delta y_{\text{top}} = \frac{1}{2}mv^2_{\text{bottom}} \\
v_{\text{bottom}} = \sqrt{2g\Delta y} = \sqrt{2(9.8 \ m/s^2)(10m)} = 14 \ m/s
\]
Kinematics Approach: For an object in free fall, its initial velocity must be zero, its displacement is 10 meters, and the acceleration due to gravity on the surface of the Earth is 9.8 m/s². Choosing down as the positive direction:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0$</td>
<td>0 m/s</td>
</tr>
<tr>
<td>$v$</td>
<td>FIND</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>10 m</td>
</tr>
<tr>
<td>$a$</td>
<td>9.8 m/s²</td>
</tr>
<tr>
<td>$t$</td>
<td>?</td>
</tr>
</tbody>
</table>

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0) \rightarrow v_y = \sqrt{v_{y0}^2 + 2a_y(y - y_0)} \rightarrow$$

$$v_y = \sqrt{2(9.8 \text{ m/s}^2)(10 \text{ m})} = 14 \text{ m/s}$$

As you can see, you reach the same conclusion regardless of approach!

5.41 Q: The diagram below shows a toy cart possessing 16 joules of kinetic energy traveling on a frictionless, horizontal surface toward a horizontal spring.

If the cart comes to rest after compressing the spring a distance of 1.0 meter, what is the spring constant of the spring?

(A) 32 N/m  
(B) 16 N/m  
(C) 8.0 N/m  
(D) 4.0 N/m

5.41 A: (A) $K = U_s = \frac{1}{2} kx^2$

$$k = \frac{2K}{x^2} = \frac{2(16 \text{ J})}{(1 \text{ m})^2} = 32 \text{ N/m}$$

5.42 Q: A child does 0.20 joules of work to compress the spring in a pop-up toy. If the mass of the toy is 0.010 kilograms, what is the maximum vertical height that the toy can reach after the spring is released?

(A) 20 m  
(B) 2.0 m  
(C) 0.20 m  
(D) 0.020 m
5.42 A: (B) The potential energy in the compressed spring must be equal to the gravitational potential energy of the toy at its maximum vertical height.

\[ U_s = U_g = mg\Delta y \]
\[ \Delta y = \frac{U_s}{mg} = \frac{0.2J}{(0.01kg)(9.8m/s^2)} = 2m \]

5.43 Q: A lawyer knocks her folder of mass \( m \) off her desk of height \( \Delta y \). What is the speed of the folder upon striking the floor?

(A) \( \sqrt{2g\Delta y} \)

(B) \( 2g\Delta y \)

(C) \( mg\Delta y \)

(D) \( m\Delta y \)

5.43 A: (A) The folder’s initial gravitational potential energy becomes its kinetic energy right before striking the floor.

\[ U_{desk} = K_{floor} \]
\[ mg\Delta y = \frac{1}{2}mv^2 \]
\[ v = \sqrt{2g\Delta y} \]

5.44 Q: A 65-kilogram pole vaulter wishes to vault to a height of 5.5 meters.

(A) Calculate the minimum amount of kinetic energy the vaulter needs to reach this height if air friction is neglected and all the vaulting energy is derived from kinetic energy.

(B) Calculate the speed the vaulter must attain to have the necessary kinetic energy.

5.44 A: (A) \( K = U_g = mg\Delta y \)

\[ K = (65kg)(9.8m/s^2)(5.5m) = 3500J \]

(B) \( K = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}} \to \]

\[ v = \sqrt{\frac{2(3500J)}{65kg}} = 10\frac{\text{m}}{\text{s}} \]
5.45 Q: The work done in accelerating an object along a frictionless horizontal surface is equal to the change in the object’s
(A) momentum
(B) velocity
(C) potential energy
(D) kinetic energy

5.45 A: (D) Due to the Work-Energy Theorem.

5.46 Q: A car, initially traveling at 30 meters per second, slows uniformly as it skids to a stop after the brakes are applied. Sketch a graph showing the relationship between the kinetic energy of the car as it is being brought to a stop and the work done by friction in stopping the car.

5.46 A: [Graph showing the relationship between kinetic energy and work done by friction]

5.47 Q: A 2-kilogram block sliding down a ramp from a height of 3 meters above the ground reaches the ground with a kinetic energy of 50 joules. The total work done by friction on the block as it slides down the ramp is approximately
(A) 6 J
(B) 9 J
(C) 18 J
(D) 44 J

5.47 A: (B) The box has gravitational potential energy at the top of the ramp, which is converted to kinetic energy as it slides down the ramp. Any gravitational potential energy not converted to kinetic energy must be the work done by friction on the block, converted to internal energy (heat) of the system.

\[ U_{\text{top}} = K_{\text{bottom}} + W_{\text{friction}} \]
\[ W_{\text{friction}} = U_{\text{top}} - K_{\text{bottom}} = mg\Delta y - K_{\text{bottom}} \]
\[ W_{\text{friction}} = (2\text{kg})(9.8\text{ m/s}^2)(3\text{ m}) - 50\text{ J} = 9\text{ J} \]
5.48 Q: Four objects travel down an inclined plane from the same height without slipping. Which will reach the bottom of the incline first?
(A) a baseball rolling down the incline
(B) an unopened soda can rolling down the incline
(C) a physics book sliding down the incline (without friction)
(D) an empty soup can rolling down the incline

5.48 A: (C) In all cases, the objects convert their gravitational potential energy into kinetic energy. In the case of the rolling objects, however, some of that kinetic energy is rotational kinetic energy. Since the physics book cannot rotate, all of its gravitational potential energy becomes translational kinetic energy; therefore, it must have the highest translational velocity.

5.49 Q: As a box is pushed 30 meters across a horizontal floor by a constant horizontal force of 25 newtons, the kinetic energy of the box increases by 300 joules. How much total internal energy is produced during this process?
(A) 150 J
(B) 250 J
(C) 450 J
(D) 750 J

5.49 A: (C). The work done on the box can be found from:
\[ W = Fd = (25N)(30m) = 750J \]
From the Work-Energy Theorem, you know that the total energy of the box must increase by 750 joules. If the kinetic energy of the box increases by 300 joules, where did the other 450 joules of energy go? It must have been transformed into internal energy!

5.50 Q: Mass \( m_1 \) sits on a frictionless surface and is attached by a light string across a frictionless pulley to mass \( m_2 \), as shown in the diagram below.

What happens to the gravitational potential energy and kinetic energy of \( m_1 \) and \( m_2 \) when \( m_2 \) is released from rest?

5.50 A: The gravitational potential energy of \( m_1 \) remains the same while its kinetic energy increases. The gravitational potential energy of \( m_2 \) decreases while its kinetic energy increases.
5.51 Q: A box of mass \( m \) is attached to a spring (spring constant \( k \)) and sits on a frictionless horizontal surface as shown below.

\[ \text{The spring is compressed a distance } x \text{ from its equilibrium position and released. Determine the speed of the box when the spring returns to its equilibrium position.} \]

5.51 A: The elastic potential energy stored in the compressed spring is converted completely to kinetic energy when the spring returns to its equilibrium position.

\[ U_s = K \rightarrow \frac{1}{2} kx^2 = \frac{1}{2} mv^2 \rightarrow v = \sqrt{\frac{k}{m}} x \]

5.52 Q: A pendulum of mass \( M \) swings on a light string of length \( L \) as shown in the diagram at right. If the mass is released from rest at an angle of theta as shown, find the maximum speed of the pendulum as a function of \( \theta, L, \) and any required fundamental constants.

5.52 A: First determine the gravitational potential energy at the mass’s highest point as shown in problem 5.27.

\[ \Delta U_g = mg\Delta y \rightarrow \Delta y = L(1-\cos \theta) \rightarrow mgL(1-\cos \theta) \]

Next, noting that this gravitational potential energy is converted to kinetic energy at the lowest point of the pendulum, solve for the velocity of the mass at the lowest point.

\[ \Delta U_g = K \rightarrow mgL(1-\cos \theta) = \frac{1}{2} mv^2 \rightarrow v = \sqrt{2gL(1-\cos \theta)} \]

5.53 Q: A ball is thrown vertically upwards. Plot the ball’s total mechanical energy, kinetic energy, and gravitational potential energy versus time on the same axes.

5.53 A:
5.54 Q: When the ball is half the distance to its peak, which of the following are true? Choose all that apply. Neglect air resistance.
(A) The ball’s velocity is half its maximum velocity \((v=v_{\text{max}}/2)\).
(B) The ball’s velocity is its maximum velocity divided by the square root of 2 \((v=v_{\text{max}}/\sqrt{2})\).
(C) The ball’s kinetic energy is equal to its gravitational potential energy.
(D) The ball’s kinetic energy is equal to half its total mechanical energy.

5.54 A: (B), (C), and (D) are all true.

5.55 Q: Andy the Adventurous Adventurer, while running from evil bad guys in the Amazonian Rainforest, trips, falls, and slides down a frictionless mudslide of height 20 meters as depicted below.

Once he reaches the bottom of the mudslide, he has the misfortune to fly horizontally off a 15-meter cliff. How far from the base of the cliff does Andy land?

5.55 A: Breaking this problem into sections, let’s first analyze Andy’s motion on the mudslide by utilizing conservation of energy to determine his horizontal velocity as he flies off the cliff.

\[ U_g = mg\Delta y = \frac{1}{2}mv^2 = K \rightarrow v = \sqrt{2g\Delta y} \rightarrow v = \sqrt{2(9.8\text{ m/s}^2)(20\text{ m})} = 19.8\text{ m/s} \]

Once Andy flies horizontally off the cliff, this becomes a projectile problem. First find Andy’s time in the air by analyzing his vertical motion, then use this time to find how far he travels horizontally before landing some distance from the base of the cliff.

\[ \Delta y = v_{y0}t + \frac{1}{2}a_yt^2 \rightarrow v_{y0} = 0 \rightarrow t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(15\text{ m})}{9.8\text{ m/s}^2}} = 1.75\text{s} \]

\[ \Delta x = v_{y0}t = (19.8\text{ m/s})(1.75\text{s}) = 34.6\text{ m} \]
5.56 Q: Alicia, a 60-kg bungee jumper, steps off a 40-meter-high bridge. The bungee cord behaves like a spring with spring constant $k=40 \text{ N/m}$. Assume there is no slack in the bungee cord.

(A) Find the speed of the jumper at a height of 15 meters above the ground.

(B) Find the speed of the jumper at a height of 30 meters above the ground.

(C) How close does the jumper get to the ground?

5.56 A: First draw a diagram of the bungee jumper, labeling the point 15 meters above the ground point A, and the point 30 meters above the ground point B.

(A) Next, use conservation of energy to solve for the kinetic energy of the jumper when she is at point A.

$$U_{g\text{TOP}} = U_{gA} + U_{sA} + K_{A} \rightarrow K_{A} = U_{g\text{TOP}} - U_{gA} - U_{sA} \rightarrow$$

$$\frac{1}{2}mv_{A}^{2} = mg\Delta y - \frac{1}{2}k(\Delta y)^{2} \rightarrow v_{A}^{2} = 2g\Delta y - \frac{k}{m}(\Delta y)^{2} \rightarrow$$

$$v_{A}^{2} = 2(9.8 \text{ m/s}^{2})(25 \text{ m}) - \frac{40 \text{ N/m}}{60 \text{ kg}}(25 \text{ m})^{2} = 73.3 \text{ m}^{2}/\text{s}^{2} \rightarrow v_{A} = 8.6 \text{ m/s}$$

(B) Follow the same strategy to determine the speed of the jumper at point B.

$$U_{g\text{TOP}} = U_{gB} + U_{sB} + K_{B} \rightarrow K_{B} = U_{g\text{TOP}} - U_{gB} - U_{sB} \rightarrow$$

$$\frac{1}{2}mv_{B}^{2} = mg\Delta y - \frac{1}{2}k(\Delta y)^{2} \rightarrow v_{B}^{2} = 2g\Delta y - \frac{k}{m}(\Delta y)^{2} \rightarrow$$

$$v_{B}^{2} = 2(9.8 \text{ m/s}^{2})(10 \text{ m}) - \frac{40 \text{ N/m}}{60 \text{ kg}}(10 \text{ m})^{2} = 129 \text{ m}^{2}/\text{s}^{2} \rightarrow v_{B} = 11.4 \text{ m/s}$$

(C) The closest the jumper gets to the ground occurs when the jumper’s speed (and therefore kinetic energy) reaches zero. Again, utilize conservation of energy.

$$U_{k\text{TOP}} = U_{s\text{bottom}} + U_{s\text{bottom}} \rightarrow \Delta U_{g} = U_{g} \rightarrow mg\Delta y = \frac{1}{2}k(\Delta y)^{2} \rightarrow$$

$$2mg = k\Delta y \rightarrow \Delta y = \frac{2mg}{k} = \frac{2(60 \text{ kg})(9.8 \text{ m/s}^{2})}{40 \text{ N/m}} = 29.4 \text{ m}$$

If $\Delta y=29.4 \text{ m}$, the jumper must be 40m-29.4m from the ground at her lowest point, or 10.6m above the ground.
Sources of Energy on Earth

So where does all this energy initially come from? Here on Earth, the energy you deal with everyday ultimately comes from the conversion of mass into energy, the source of the sun’s energy. The sun’s radiation provides an energy source for life on Earth, which over the millennia has become the source of fossil fuels. The sun’s radiation also provides the thermal and light energy that heat the atmosphere and cause the winds to blow. The sun’s energy evaporates water, which eventually recondenses as rain and snow, falling to the Earth’s surface to create lakes and rivers, with gravitational potential energy, which is harnessed in hydroelectric power plants. Nuclear power also comes from the conversion of mass into energy. Just try to find an energy source on Earth that doesn’t originate with the conversion of mass into energy!
Test Your Understanding

1. Design and perform an experiment which examines how a force exerted on an object does work on the object as it moves through a distance.

2. Is it possible to do work on an object without changing the object’s energy? Explain why or why not with examples.

3. Humpty Dumpty falls off the wall onto the ground and breaks. If Humpty Dumpty initially has gravitational potential energy up on the wall, describe what happened to this energy, keeping in mind the law of conservation of energy.

4. Why can a single object have only kinetic energy, while potential energy requires interactions among objects or systems? Explain with real-world examples.

5. Create a conservation of energy problem that utilizes at least three different types of energy. Solve it.

6. In your own words, explain what is meant by open and closed systems. Give examples of how both energy and linear momentum are conserved in these systems.

7. An object is dropped from a given height h at the same time an identical object is launched vertically upward from ground level. Determine the initial velocity of the object launched upward such that the two objects collide at height h/2. Having already answered this problem using kinematics at the end of Chapter 3, now answer using a conservation of energy approach. Which approach is easier?